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Determination of Young's Modulus of Elasticity by an Optical Method.

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Abstract:

An interferometric method is used to investigate the effect of controlled stress on the optical behaiour of a transparent isotropic acrylic and glass samples. Hence, the stress optical coefficient and Young's modulus of elasticity are evaluated. The inducded birefringence as well as its dispersion is measured.

1. Introduction

Normally transparent isotropic substances could be made optically anisotropic by the application of mechanical stress. The phenomenon is variously known as mechanical birefringence, Photoelasticity or stress birefringence. Under compression or tension the material takes on the properties of a negative or a positive uniaxial crystal, respectively. In either case the effective optic axis is in the direction of the stress, and the induced birefringence is proportional to the stress. Clearly, if the stress is not uniform over the sample, the birefringence and retardation will be imposed on the transmitted wave. [1]

Photoelasticity is a common method for stress analysis of specimen subjects to loads [2,3]. A copy, with reduced scale, of the specimen under investigation is made. Transparent materials like glass, gelatin, bakelite, acrylic, polymers and epoxy can be used for making such models. The stressed models are enclosed between two crossed polarizers and illuminated with a monochromatic light. The stressed regions in the model rotate the plane of polarization of light and a visible interference pattern is produced. The direction and size of the stresses may be deduced from this interference pattern. [4]

Photoelasticity is a well-established technique for stress analysis, and it has a wide range of industrial and research applications. Several methods of analyzing photoelastic fringe pattern by means of phase-measuring algorithm for extraction of the isochromatic of photoelastic fringe patterns. The isochromatic phase map obtained with this algorithm has been well suited for a full separation of the stress components in a sample. The Young's modulus of amorphous polymers around the glass-to rubber transition zone can be separated into two component functions (R and G) through a modified stress-optical rule. The two component functions were compared among more than ten polymers, and their relation to chemical structure was studied [5]. A controlled stress was applied to a transparent sheet of plastic. The resulting interference fringes were recorded and digitized. These data were fed into a computer with proper formula to calculate the coefficients of the curve, which describes the applied stress distribution.

In this work, experimental measurement of the effect of controlled stress on isotropic materials to induce birefringence is considered. Some constants such as Young's modulus of elasticity and stress optical coefficient are evaluated.

2. Theory of induced birefringence

When a polymeric material is deformed by stress, birefringence arises as well as strain. The relation between birefringence and stress has long been a subject of important optical studies. Deformation birefringence arises as a result of a change of bond angles and/or bond length, or a change in packing (a change in the lattice spacing) by an external deformation. For polymer melts and concentrated solutions, the anisotropic part of the refractive index tensor, n (t) is proportional to that of the stress tensor σ (t). This empirical rule is called the stress-optical rule SOR. The SOR for tensile deformation can be written as follows [6].

$$\Delta n (t) = C\sigma (t) \qquad (1)$$

Here, $\Delta n(t)$ is the birefringence and σ (t) is the tensile stress. The coefficient of proportionality "C", called the stress-optical coefficient depends upon the properties of the material. The validity of the rule has been examined for many polymeric systems. From a theoretical point of view, the SOR indicates that the molecular origin of the stress as well as the birefringence is attributed to the orientation of the chain. Many molecular theories can predict the validity of the rule in rubbery or liquid states.

Suppose that the stressed model is inserted between two crossed polarizers and illuminated with monochromatic light. The induced birefringence of the model causes the polarized light (from the polarizer) to emerge refracted into two orthogonal planes. The light propagates in the same or different directions but with different velocities and this produces a phase shift between the transmitted light waves. When the analyzer recombines the waves, regions of stress where the differences of the waves are even multiples of π appear dark. The regions of the stress where the phase differences are odd multiples of π appear bright .The difference in phase between the two waves is given by

$$\varphi = (2\pi/\lambda)\, t\, (n_o - n_e) \qquad (2)$$

Where n_o and n_e are the refractive indices in the parallel and perpendicular directions to the applied stress. The stress model produces a relative retardation that is equal to:

$$R = 2\pi\, t\, C\, (\sigma_1 - \sigma_2)\, /\lambda \qquad (3)$$

Where $(\sigma_1 - \sigma_2)$ is the difference in the in-plane principal stresses. The transmitted intensity behind the second polarizer is [7]:

$$I = E^2 (\sin 2\beta)^2 (\sin R/2)^2 \qquad (4)$$

Where β is the angle beween the axis of the polarizer and the dirction of the first principal stress in the model. The contours of equal intensity represent contours of equal stress. There are two conditions for the transmitted intensity of light to be zero. The first case is when $\beta = 0, \pi/2, 3\pi/2, \ldots,$. In this case, the axis of the polarizer coincides with any of the two in-plane principle stress directions in the model. The loci of points of equal stress will therefore be dark bands that are called isoclinic, which means that it is of equal inclination. The second case is when $R = 0, 2\pi \ldots m(2\pi)$, where "*m*"is the order of interference and an integer number from zero to infinity. It is equal to the number of wavelengths by which the two interfering waves are shifted from each other due to the applied stress. Loci of equal values of "*m*" are also black curves and are known as the isochromatic fringes which are curves of equal principal stress difference.

3. Optical set-up and Experiment

The optical set-up for recording the interferogram of strain distribution is shown in fig.1. S_0 is a monochromatic source of light. G is a glass filter, L_1 is a condensing lens with focal length 3*cm* to form a minimized image of the source on the pinhole, C is a pinhole to increase the spatial coherence of the light source. L_2 is a collimating lens of focal length 5*cm* to produce a parallel beam of light. N_1 is a linear polarizer to give a linearly polarized light. S is the sample under stress to be investigated, which is a transparent isotropic material such as Acrylic, and glass. N_2 is a linear polarizer. L_3 is a condensing lens with long focal length 15*cm* to form an image on a Fortepan photographic plate F to record the interference pattern. A hydraulic press HY for the compression on the sample is used.

Without a compressive force there are no interference fringes in the field of view, where the polarizer and the analyzer are crossed. By applying a force on the sample the interference fringes start to appear. The number of fringes increases and its shape changes as the magnitude of the applied force increases. As the applied force decreases the number of fringes decreases till the fringes disappear when there is no force applied.

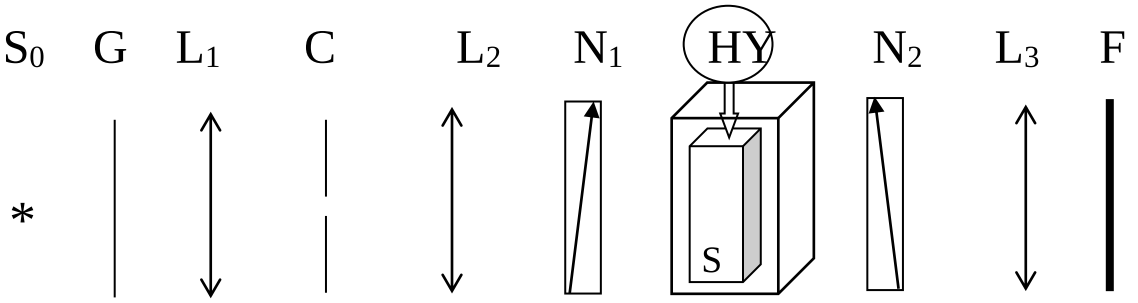


Fig 1: Optical set-up for investigating the stress distribution in a sample.

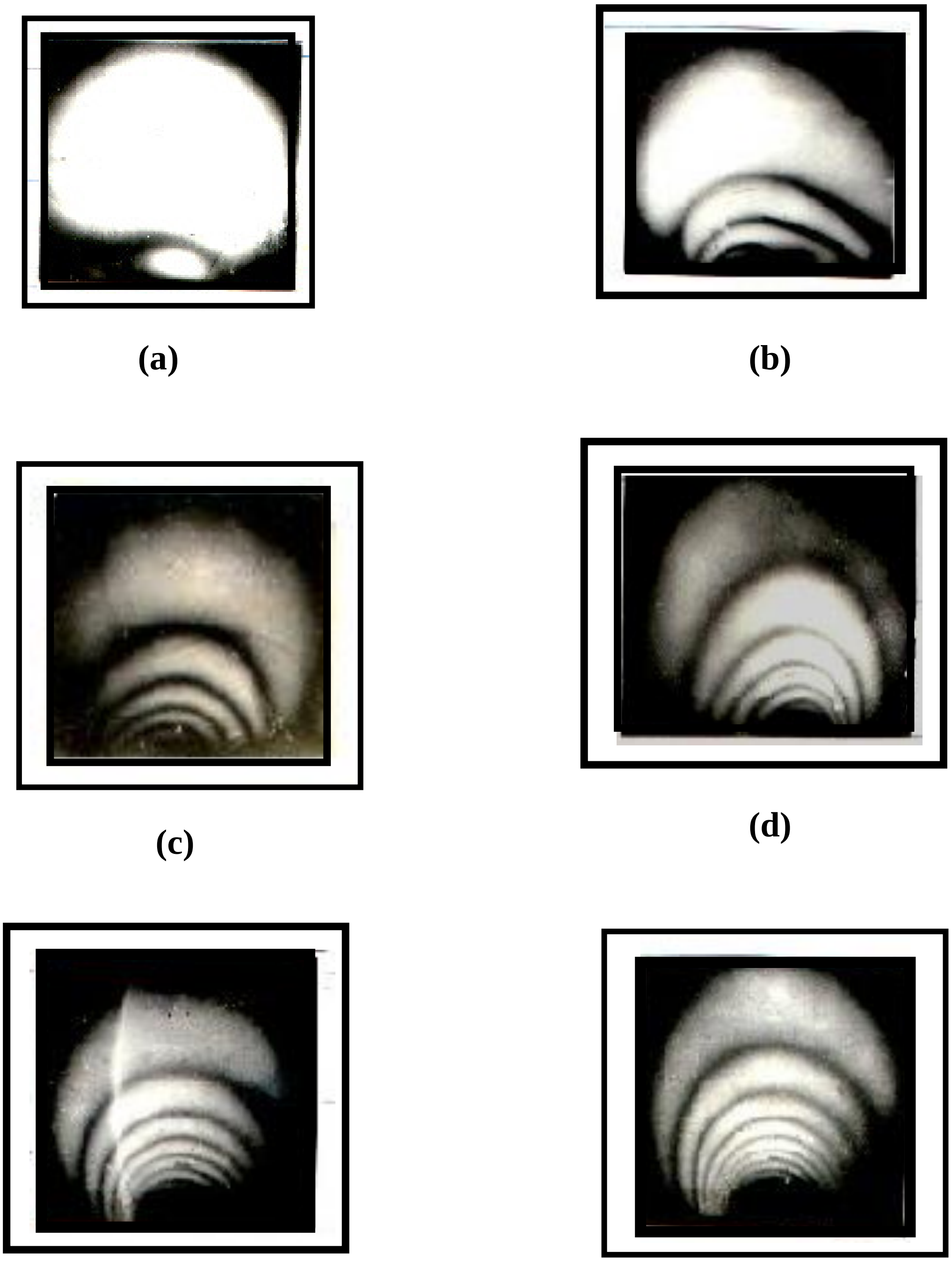
An acrylic plate (5cm length and 1.82cm thick.) as a sample is placed between two crossed polarizers under a hydraulic press and compressing it with different stresses, the interference fringes with different specified stresses are shown in plate1.

4. Strain distribution and Young's modulus

A method of fitting quadratic forms, by using a PC-computer, for analyzing the resulting interference fringes is adopted to find the distribution of the applied stress as follows. The fringes are numbered according to their orders of appearance.

Then, each fringe is traced with equal steps (the steps are chosen approximately equal to the mean fringe separation) to find the positions of x and y on the fringe with reference to the photographic plate F. The (x, y) indices for each step are recorded together with the order of appearance assigned to that fringe. These data are fed to the computer. A least squares algorithm for fitting quadratic forms is designed. The algorithm is then used to determine the coefficients and the residual variance for the fitting of a quadratic equation in the form:

$$Y = A + BX + CX^2. \tag{5}$$



(a) **(b)**

(c) **(d)**

(e) **(f)**

Plate 1 The interference fringes obtained due to variable stresses on acrylic sample. Stress equal to (a) 49.0 x 10^4N/m^2. (b) 58.8 x10^4N/m^2 (c) 68.6 x10^4N/m^2. (d) 78.4 x10^4N/m^2. (e) 88.2 x10^4N/m^2.

From the coefficients, A, B and C, the shape of the interference fringes and consequently the shape of the applied stress is deduced. These coefficients indicate that the fringes are of the shape of hyperbolas [4]. This stress is distributed in the form of hyperbolas. The induced birefringence can be obtained as:

$$t\,\Delta n = m\,\lambda \qquad (6)$$

Where t is the thickness of the material used, *m* is the order of appearance of the fringe that exists due to applying the stress and λ is the wavelenth of the light used.

The value of the constant A represents the displacement of the fringes in the direction of the applied stress (y-axis) at x = 0 and the values of the constants B and C are useful to find the stress distribution. The coefficient A in eq.(5) is directly proportional to the values of the applied stresses on the material used. Since the strain is the fractional change in length of the specimen therefore the ratio of the reduction in the length of the sample, ΔL, to the original length L is the strain of the material. This ratio is a dimensionless quantity. The value of ΔL is found from:

$$\Delta L = (A / \Delta A)\,\lambda \qquad (7)$$

Where ΔA is the average fringe separation. Table shows the fitting parameters of eq.(5) for the acrylic sample subjected to different specified stresses. This table illustrates the strain distribution inside the sample. Fig.2 represents the strain distribution in this sample by using sodium lamp of wavelength 589.3 nm.

Table 1: *Fitting parameters of eq.(5) for acrylic sample subjected to different specified stresses by using sodium lamp of wavelength 589.3 nm.*

Applied stress (	Fringe or	Fitting parameters		
	m	A	B	C
58.8	1	0.3607919	3.60057E-17	-1.3640308
68.6	2	0.57414719	-1.48243E-17	-1.5658159
78.4	3	0.69117074	1.660833E-17	-1.3444371
88.2	4	0.94880092	-9.13052E-17	-1.370741
98.0	5	1.2588483	-2.10517E-16	-1.3822537

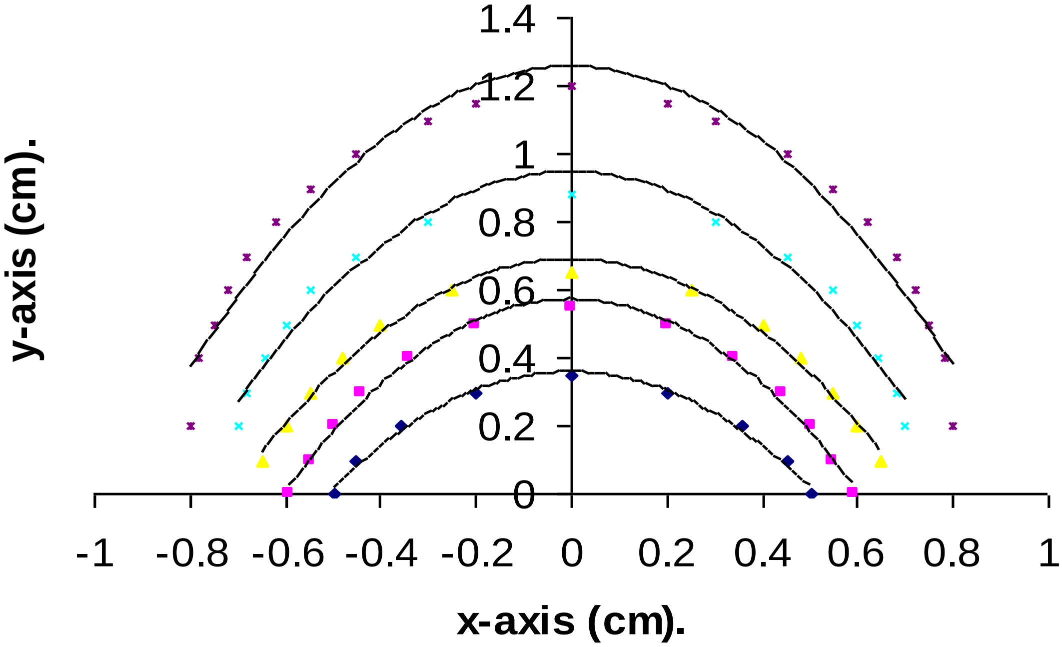


Fig 2: The strain distribution for acrylic sample (5cm length and 1.82 cm thick.) subjected to stress of 98x 10^4 N /m^2 by using sodium lamp of wavelength 589.3nm.

In this figure the continuous lines are those of fitting and the doted are the experimental values.

A mercury spectral lamp of wavelengths 578.0, 546.1 and 435.8 nm is used as a source. In addition to the mercury lamp the spectral sodium lamp and He-Ne laser source with wavelengths of 589.3, 632.8 nm, are respectively used. The relation between different applied stresses and the produced strain are illustrated in fig.3. From the definition of Young's modulus, which is equal to the ratio between the applied stress and the strain, its value can be obtained. Since these figures are straight lines, it is fitted to the linear function $y = a x + b$ where y axis represents the stress and x axis the strain, a and b are constants, where a is the Young's modulus and b is the intercept with the y-axis. The value of b deduces the threshold value of the stress, which should be applied in order to attain detectable strain.

The values of Young's modulus corresponding to different wavelengths for acrylic sample are shown in table 2. According to this table the Young's modulus depends on the frequency of the light used, actually it should not. Therefore the value at zero frequency is our aim. The values of Young’s modulus corresponding to different wavelengths for acrylic sample are shown in table 2. According to this table the Young’s modulus depends on the frequency of the light used, actually it should not. Therefore the value at zero frequency is our aim. The relation between Young’s modulus and the frequency is shown in fig.4. The true Young’s modulus is that corresponding to zero frequency which is (26.368 N / m^2). This value is in good agreement with the published data [8,9].

The same work is applied on a sample of crown glass and Young’s modulus is found for it as in fig.5. Young’s modulus for crown glass sample is 21.4855 x 10^9N/m^2.

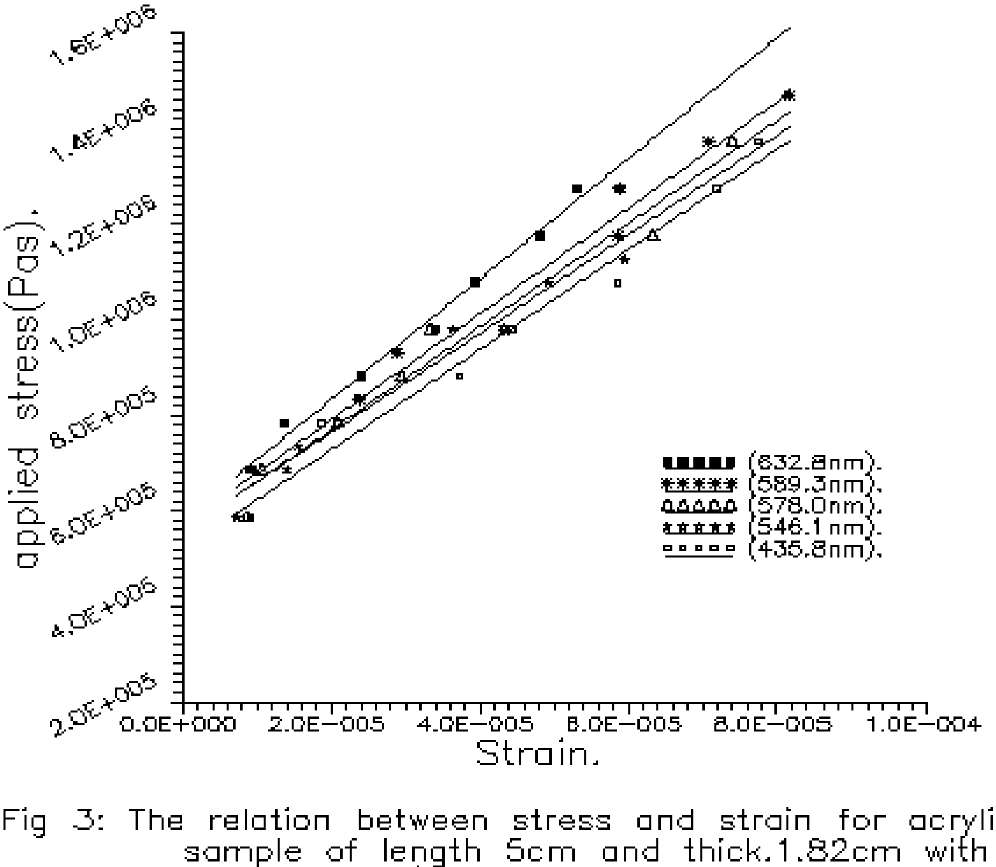


Fig 3: The relation between stress and strain for acrylic sample of length 5cm and thick.1.82cm with differet wavelengths (632.8, 589.3, 578.0, 546.1, 435.8 nm)

Table 2: *The values of Young's modulus for acrylic sample with different wavelengths or frequencies (1/λ).*

Wavelength λ (nm)	1/λ (nm^{-1})	Young's modulus [(Pas)]
632.8	0.001580278	12.5344
589.3	0.001696928	11.0721
578.0	0.001730103	10.7730
546.1	0.001831166	10.4722
435.8	0.002294630	10.3334

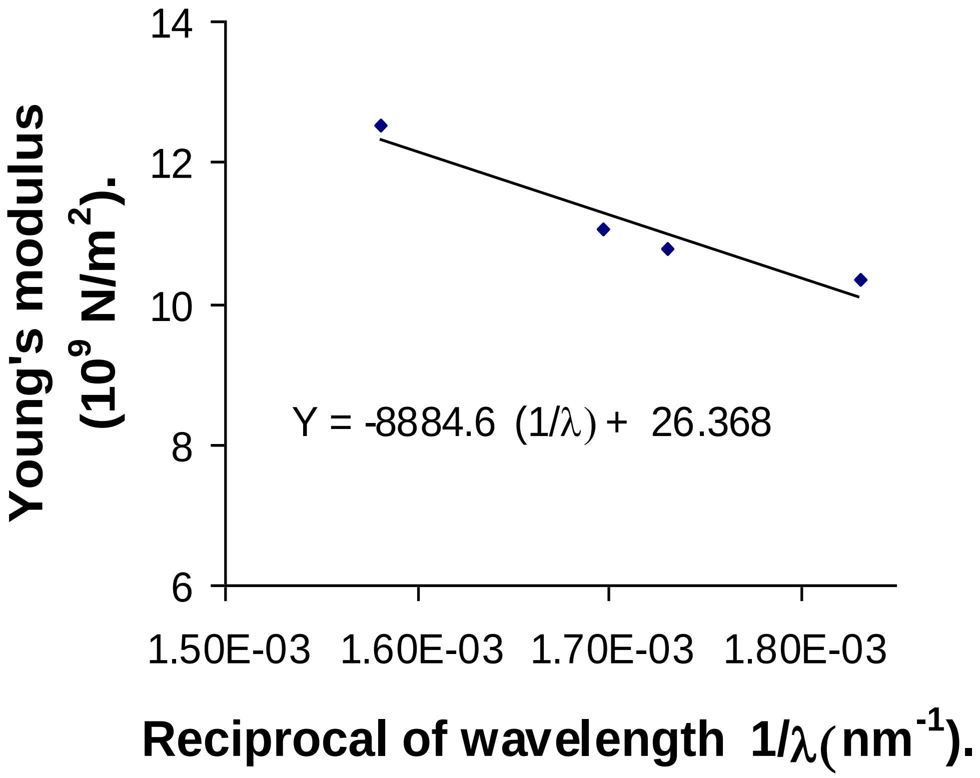


Fig 4: The relation between Young's modulus for acrylic and the frequency of the radiation used.

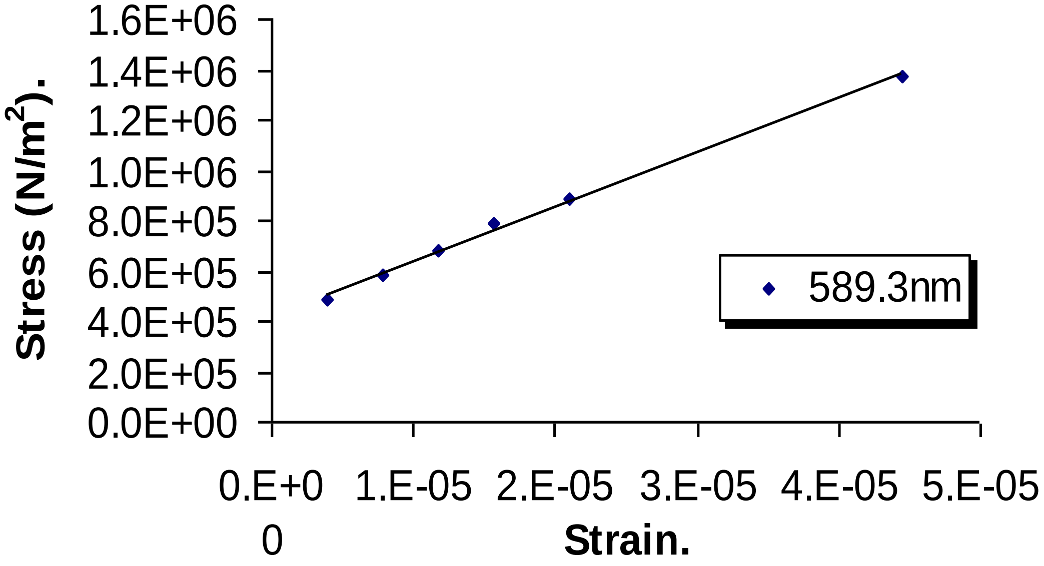


Fig 5: The relation between stress and strain for Glass sample of length 5cm.with sodium lamp of wavelength (589.3nm).

5. Measurement of birefringence and its dispersion

The induced birefringence "Δn" is obtained by using eq. (6) where the order of interference m is the order of appearance of the fringe, which is equal to A/ΔA. Since $Strain = \frac{\Delta L}{L} = \frac{A}{\Delta A} \cdot \frac{\lambda}{L}$, so that by substituting in eq.(6) for λ, Δn is obtained with different strains at constant stress for the different wavelengths as in fig.3. Dispersion relation is obtained for acrylic sample as shown in fig.6, which represents the relation between the birefringence and wavelengths of light used at constant stress. Birefringence is inversely proportional to wavelength and the relation is in good agreement with normal dispersion of Cauchy equation for natural birefringent materials, in the form

$$\Delta n = a + b / \lambda^2 \qquad (8)$$

Where least squares fitting to this equation gives; $a = 2.935 \times 10^{-5}$,and $b=26.665$ (nm^2).

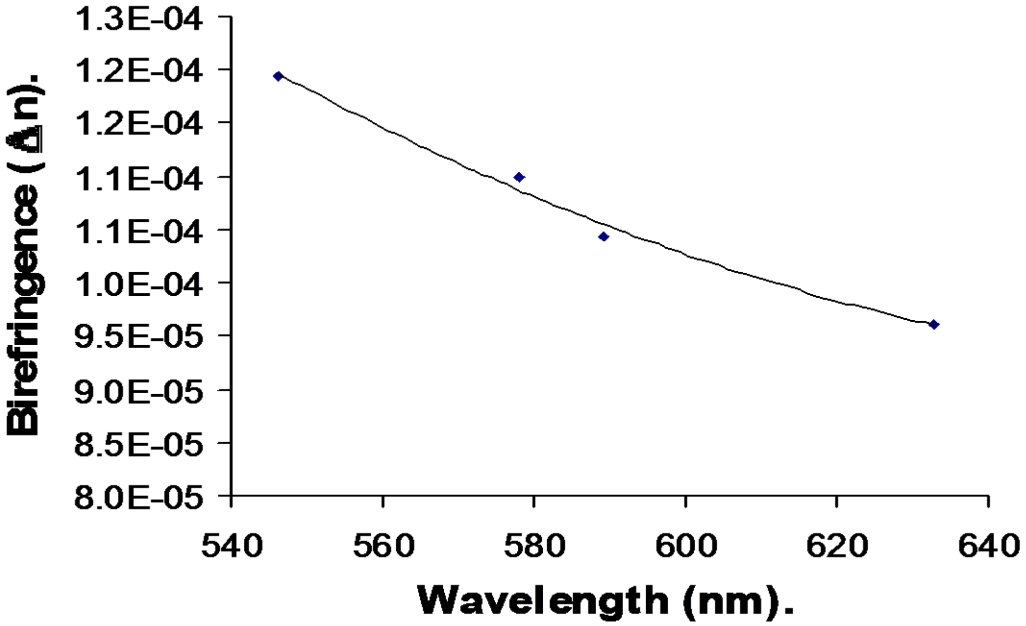


Fig 6: The relation between birefringence and the wavelength of light used for acrylic sample of length 5cm at constant stress of (98.0x10^4 N/m^2).

From the relation between applied stress and the induced birefringence in eq. (1), the value of the stress optical coefficient "C" is obtained as the slope of the relations given in fig.7. The value of the stress optical coefficient "C" for the sample of acrylic for different wavelengths is shown in table 5. These values are in good agreement with the published data [8, 10].

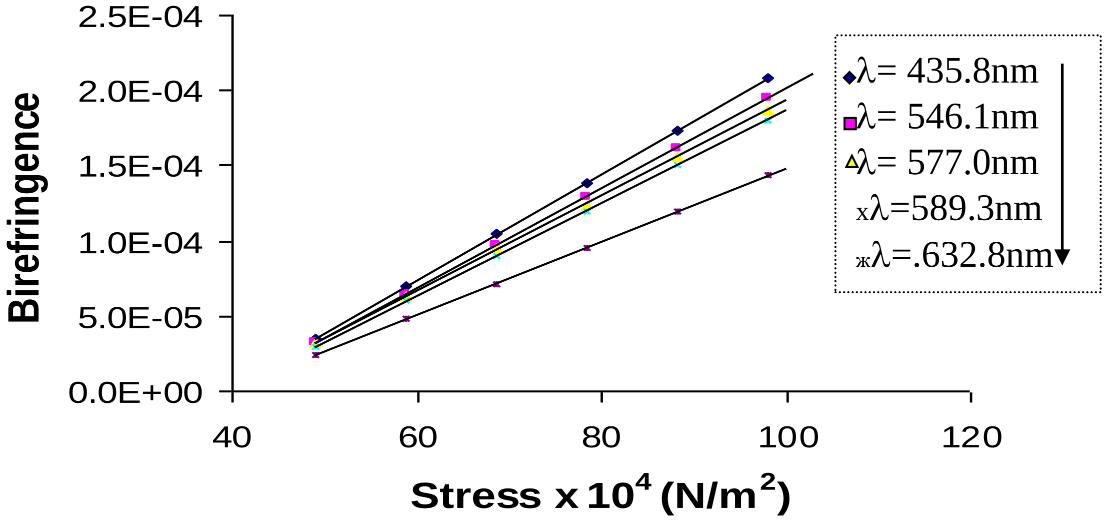


Fig. 7: The relation between applied stress and birefringence for an acrylic sample of thickness 1.82 cm with different wavelengths.

Table 3: *The stress optical coefficient C for acrylic sample with different wavelengths.*

Wavelength λ (nm)	Stress optical coeffi (Brewster)*.
632.8	0.0354802
589.3	0.0330399
578.0	0.0324064
546.1	0.0306179
435.8	0.0244337

* Browster = 10^{-12} (m^2/N)

6. Conclusion

Simple and accurate optical system is used to evaluate some optical constants such as the induced birefringence, Young's modulus of elasticity and stress-optical coefficient for some isotropic materials like the Acrillic and glass by mechanical method. Its dispersion across the visible region is studied. The distribution of the fringes makes it possible to assess the distribution of stresses inside the plate. This underlies the optical method of studying stresses (photoelastic stress analysis). A model made from a transparent isotropic material is placed between crossed polarizers. The model is subjected to the action of loads similar to those, which the article itself will experience. The pattern observed in transmitted white light makes it possible to determine the distribution of the stresses and also to estimate its magnitude. The induced birefringence produced by stress on an isotropic material decreases by increasing wavelength and Cauchy's dispersion function is achieved. The dispersion increases by increasing applied stress upon the sample. From the relation between the applied stress and the resulting strain which is build upon the displacement in the fringe, Young's modulus is evaluated by a simple and accurate method. The stress optical coefficient is obtained also from the relation between the induced birefringence and the applied stress upon the material used with different wavelengths. The stress optical coefficient depends The stress optical coefficient is on the wavelength of the light and the material used.

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